The University of Burdwan

SYLLABUS FOR 3-YEAR DEGREE/4-YEAR HONOURS IN MATHEMATICS

Under Curriculum and Credit Framework

for

Undergraduate Programmes (CCFUP) as per NEP, 2020

With effect from 2023-2024

Preamble

Undergraduate (UG) Programme is of either 3 or 4-year duration, with multiple entry and exit points and re-entry options, with appropriate certifications such as:

- UG Certificate after completing 1 year (2 Semesters) of study in the chosen fields of study
- UG Diploma after 2 years (4 Semesters) of study
- Bachelor's Degree after 3 years (6 Semesters) programme of study
- Bachelor's Degree (Honours) after 4 years (8 Semesters) programme of study
- Bachelor's Degree (Honours with research) after 4 years (8 Semesters) programme of study, if the students complete a rigorous Research Project/ Dissertation in their major area(s) of study in the 4th year of a Bachelor's Degree.

The courses offered at the UG level are grouped into eight broad categories which along with the minimum credit requirements are as follows.

Broad Category of Course	**Credit requirement**		
	3-year UG Degree Prog.	**4-year UG Honours Prog.**	
		With RP	**Without RP**
Major (Core)	64	94	106
Minor	28	36	36
Multidisciplinary	09	09	09
Ability Enhancement Courses (AEC)	08	08	08
Skill Enhancement Courses (SEC)	09	09	09
Value Added Courses common for all UG students	08	08	08
Summer Internship	02	02	02
Research Project/Dissertation*	---	12	---
Total	128	178	178

*Honours students not undertaking research will pursue three major courses (each is of 4 credits) for 12 credits in lieu of a Research Project/Dissertation.

SEMESTER WISE & COURSE WISE CREDIT DISTRIBUTION STRUCTURE UNDER CCFUP AS PER NEP, 2020 FOR 3-YR. DEGREE AND/OR 4-YR. HONOURS PROGRAMME(S)

Semester	Course Type with Code	Level	Name of the Course	Credit	Lect.	Tuto.	Pract./Viva-voce	Full Marks	Distribution of Marks		
									Theory	Pract./ Viva-voce	Internal Assessment
III	Major/DS Course (Core) Code: MATH3011	200-299	Real Analysis I	5	4	1	0	75	60	0	15
	Major/DS Course (Core) Code: MATH3012	200-299	Linear Algebra	5	4	1	0	75	60	0	15
	Minor Course Code: 3021	200-299	Intermediate Level Course (Voc. Edn. &Trng.)	4	3	1	0	75	60	0	15
	Multi/Interdisciplinary Code: MATH3031		Calculus	3	2	1	0	50	40	0	10
	Ability Enhancement Course (AEC)[L_1-2 MIL] Code: 3041		MIL(L_1):Arabic/ Bengali/ Hindi/ Sanskrit/ Santali/ Urdu or Equvlnt. Course from SWAYAM /Any other UGC recognized platform	2	2	0	0	50	40	0	10
	Skill Enhancement Course (SEC) Code: MATH3051		Mathematical Modelling	3	2	1	0	50	40	0	10
	Total			**22**				**375**			

Semester	Course Type with Code	Level	Name of the Course	Credit	Lect.	Tuto.	Pract./Viva-voce	Full Marks	Distribution of Marks		
									Theory	Pract./ Viva-voce	Internal Assessment
IV	Major/DS Course (Core) Code: MATH4011	200-299	Metric Spaces	5	4	1	0	75	60	0	15
	Major/DS Course (Core) Code: MATH4012	200-299	Group Theory & Ring Theory	5	4	1	0	75	60	0	15
	Major/DS Course (Core) Code: MATH4013	200-299	Multivariate Calculus & Tensor Calculus	5	4	1	0	75	60	0	15
	Minor Course (For the students chosen Mathematics as minor subject in Sem-I or II) Code: MATH4021	200-299	Ordinary Differential Equations	4	3	1	0	75	60	0	15
	Minor Course (Other than Mathematics) Code:	200-299		4	3/3	1/0	0/1	75	60/40	0/20	15
	Ability Enhancement Course (AEC)[L_2-2] Code: ENG4041		English or Equvlnt. Course from SWAYAM	2	2	0	0	50	40	0	10
	Total			**25**				**425**			

Students exiting the programme after securing 87 credits will be awarded UG Diploma in the relevant Discipline /Subject provided they secure additional 4 credits in skill based vocational courses offered during summer term of fourth semester.

Objectives

- To impart teaching so that the students could develop higher-order thinking capacities about the fundamental aspects of mathematics.
- To train the students with mathematical knowledge and computational techniques so that they can deal with the problems faced in different walks of life.
- To impart sophisticated mathematical skills so that students can undertake self-employment initiatives.
- To make the students capable of pursuing research work in various emerging fields of mathematics and its applications.

Pre-requisite

For major, minor and skill development courses, the students should possess the knowledge on the mathematics courses at (10+2) level. For multidisciplinary courses the students should possess the knowledge on the mathematics courses at secondary level.

Programme Outcomes

- Development of critical thinking for solving complex problems.
- Skills to characterise problems, formulate a hypothesis, evaluate and validate outcomes, and draw reasonable conclusions thereof.
- Development of the effective scientific and technical communications in both oral and written forms.

Programme Specific Outcomes

- Understanding the fundamental axioms in mathematics, and capability of developing ideas based on them.
- Development of mathematical reasoning and an understanding of the underlying fundamental structures of mathematics (i.e., sets, relations and functions, logical structure), and the relationship among them.
- Motivation for research studies in mathematics and related fields with real life applications.
- Knowledge in a wide range of mathematical techniques and applications of mathematical methods/tools in other scientific and engineering domains.
- Nurturing problem-solving skills, thinking, creativity through assignments, tutorials.
- Preparing for various competitive examinations at the national and international levels.

DETAILED SYLLABUS

SEMESTER – III

MAJOR COURSES

Course Code: MATH3011
Course Name: Real Analysis I (Credit: 5, Marks: 75)
Total Hours: Lecture -60, Tutorial – 15

Objectives:

To familiarize the students with the fundamental concepts of Real Analysis such as countable set, uncountable set, Archimedean property, completeness property, open set, closed set, compact set in $\mathbb{R}$. Also, to present the concepts of sequence of real numbers, series of real numbers, limit and continuity of real valued functions defined on subsets of $\mathbb{R}$.

Learning outcomes:

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. Order property, Archimedean property, completeness property of $\mathbb{R}$.
ii. Countable set, uncountable set, limit point, interior point, open set, closed set, compact set in $\mathbb{R}$.
iii. Sequences, subsequence and series of real numbers.
iv. Limit, continuity and uniform continuity of real valued functions defined on subsets of $\mathbb{R}$ including their interrelationship.

Skills: The students would be able to

i. Characterize subsets of $\mathbb{R}$ which are open, closed, countable, uncountable, compact.
ii. Characterize sequences and subsequences in $\mathbb{R}$ which are convergent or divergent.
iii. Determine which infinite series of real numbers is convergent and which is not by using various test in their course.
iv. Calculate limit of real valued functions defined on subsets of $\mathbb{R}$.
v. Characterize real valued functions defined on subsets of $\mathbb{R}$ which are discontinuous, which continuous and which are uniformly continuous.

General Competence: The students would gain

i. Some fundamental concepts of real analysis which help them to learn all the branches of mathematics smoothly.
ii. Analytical and reasoning skills, which improve their thinking power.

Contents:

Review of algebraic and order properties of $\mathbb{R}$, idea of countable sets, uncountable sets and uncountability of $\mathbb{R}$. Bounded above sets, bounded below sets, bounded sets, unbounded sets. Supremum and infimum. Completeness property of $\mathbb{R}$ and its equivalents. The Archimedean Property, dense sets in $\mathbb{R}$. Density of rational and irrational numbers in $\mathbb{R}$. Intervals, ε - neighbourhood of a point in $\mathbb{R}$, interior points of a set, open set, limit point of a set, isolated points, derived set, closed set. Interior, exterior, frontier and boundary of a set. Bolzano – Weierstrass Theorem for sets. Compact Sets in $\mathbb{R}$, Heine – Borel Theorem. **[L-20H & T-5H]**

Sequences of real numbers, bounded and unbounded sequences, convergent sequence, limit of a sequence and related Theorems. Monotonically increasing and decreasing sequences, relevant theorems, subsequences, theorems on monotone subsequence, Bolzano – Weirstrass theorem for sequences. Cauchy sequences, Cauchy's convergence criterion, lim sup, lim inf and associated theorems. **[L-13H & T-3H]**

Infinite series of real numbers, convergence and divergence of infinite series, Cauchy's convergence criterion, Abel's – Pringsheim's Theorem. Tests for convergence: Comparison Tests, D' Alembert's Ratio Test (Ratio Test), p - series, Cauchy's root test, Raabe's test, Gauss's test, Logarithmic test, De Morgan and Bertrand test, Integral test, Cauchy's condensation test. Alternating series, Leibnitz's test, Absolute and conditional convergence. Riemann's rearrangement theorem (statement only).
[L-12H & T-3H]

Limit of a function ($\varepsilon-\delta$ definition), sequential criteria for limits, divergence criteria, algebra of limits & theorems, infinite limits and limits at infinity. Continuous functions, sequential criteria for continuity and discontinuity. Algebra of continuous functions. Continuous functions on an interval, Bolzano's theorem on continuity, intermediate value theorem, fixed point theorem. Uniform continuity, non-uniform continuity criteria, theorems on uniform continuity. **[L-15H & T-4H]**

Suggested Books:

Text Books:

1. Introduction to Real Analysis - R.G. Bartle and D.R. Sherbert, (John Wiley and Sons (Asia) Pvt. Ltd., Singapore)(3rd Ed.,).
2. Mathematical Analysis- Tom M. Apostol, (Narosa Publishing House, 1981).
3. Calculus and mathematical Analysis- S. Goldberg.

Reference Books.

1. Introduction to Real Analysis - S. K. Mapa, (Sarat Book Distributors, Kolkata – 73).
2. Real Analysis - B.K. Lahiri & K.C. Roy, (World Press, Calcutta, 1988).
3. An Introduction to Analysis (Differential Calculus) - R.K. Ghosh & K.C. Maity, (New Central Book Agency (P) Ltd., Kolkata – 700009).
4. Mathematical Analysis - S. C. Malik & Savita Arora, (New Age International Publishers).

Course Code: MATH3012

Course Name: Linear Algebra (Credit: 5, Marks: 75)

Total Hours: Lecture -60, Tutorial – 15

Objectives:

To present a systematic introduction of the fundamental concepts of Linear Algebra and some of its applications.

Learning outcomes:

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence. Linear algebra is a basic course in almost all branches of science.

Knowledge: The students would gain knowledge about

i. vector space and its dimension.
ii. linear transformation, transpose of a linear transformation and their matrix representation.
iii. system of linear equations and various methods to solve them.
iv. eigenvalues, eigenvectors, diagonalizability, canonical forms of a matrix.
v. inner product space, orthogonalization process, normal and self-adjoint operators.

Skills: The students would be able to

i. compute a basis and dimension of a vector space.
ii. compute matrix representation of matrix and its transpose,
iii. compute the characteristic polynomial, minimal polynomial, eigen value, eigen vector of a matrix as well as of a linear operator and use them in the basic diagonalization result.
iv. find canonical forms of a matrix
v. solve system of linear equations using Gaussian elimination method and matrix inversion method
vi. compute orthogonality of vectors in an inner product and applying Gram−Schmidt orthogonalization process they will obtain an orthonormal basis of an inner product space.

General competence: The students would gain

i. fundamental concepts of vector space, linear transformation, matrix representation of a linear transformation, solution methods of a system of equations, canonical forms of a matrix, diagonalization, orthogonalization, which will be useful for further studies in every branch of mathematics.
ii. analytical and reasoning skills, which improve their thinking power.

Contents:

Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces, extension, deletion and replacement theorems. **[L-8H & T-2H]**

Linear transformations, null space, range, rank and nullity of a linear transformation, matrix representation of a linear transformation, algebra of linear transformations, transpose of a linear transformation and matrix representation of the transpose of a linear transformation, Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, change of coordinate matrix.

[L-12H & T-3H]

Elementary operations on matrices, row reduction and echelon forms of a matrix, rank of a matrix, characterization of invertible matrices using rank. Eigenvalues, Eigenvectors and Characteristic Equation of a matrix. Cayley-Hamilton theorem and its use in finding the inverse of a matrix.

[L-12H & T-3H]

System of linear equations, the matrix equation $Ax = b$, necessary and sufficient condition for consistency of a linear non-homogeneous system of equations, solution of systems of linear equations using Gaussian elimination method and matrix inversion method, solution sets of linear systems, applications of linear systems. **[L-8H & T-2H]**

Eigen spaces of a linear operator, diagonalizability, invariant subspaces, the characteristic polynomial and the minimal polynomial of a linear operator, diagonalization, Jordan canonical forms.

[L-12H & T-3H]

Inner product spaces and norms, Gram-Schmidt orthogonalization process, orthogonal complements and projections. **[L-4H & T-1H]**

Bilinear form, matrix associated with a bilinear form, quadratic form, rank, signature and index of a quadratic form, Sylvester's law of inertia (statement only), reduction of a quadratic form to normal form. **[L-4H & T-1H]**

Suggested Books:

Text books:

1. Linear Algebra - S. H. Friedberg, A.J. Insel & L.E. Spence, 4th edition (Prentice Hall of India pvt., 2004).
2. Linear Algebra - K. Hoffman, R. Kunze, 2nd edition (Pearson Education Limited, 2016).
3. Higher Algebra: Abstract and Linear - S. K. Mapa, (Levant Books, 2020) (1st Edition).

Reference books:

1. Linear Algebra- A Geometric Approach - S. Kumaresan, (Prentice Hall of India).
2. Linear Algebra - A. R. Rao & P. Bhimasankaram, 2nd Edition (Hindustan Book agency, 2000).
3. Topics in Algebra - I. N. Herstein, 2nd edition (John Wiley & Sons Inc (Sea) Pte Ltd, 2017).
4. Linear Algebra - S. K. Berberian (Oxford University Press, 1992).
5. Linear Algebra - S. Lang, (3rd edition) (Springer, 1987).
6. Basic Abstract Algebra - P. B. Bhattacharya, S. K. Jain & S. R. Nagpaul,, (2nd edition) (Cambridge University Press, 2014).
7. Linear Algebra Done Right - Sheldon Axler, (3rd edition) (Springer, 2015).

MULTIDISCIPLINARY COURSES

Course Code: MATH3031
Course Name: Calculus (Credit: 3, Marks: 50)
Total Hours: Lecture - 30, Tutorial – 15

Objectives:

To introduce the concepts of differential and integral calculus and their applications. Also, to give students a basic idea of ordinary differential equation.

Learning outcomes:

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

(i) Limits and continuity of a function.
(ii) Derivative of a function
(iii) Integration of a function
(iv) applications of differential and integral calculus
(v) first order ordinary differential equations.

Skills: The students would be able to

(i) find the limits of a function,
(ii) check the continuity of a function,
(iii) find the derivatives of a real function
(iv) find the maximum or minimum values of a function
(v) integrate standard algebraic and trigonometric functions.
(vi) find the area enclosed by a curve.

General competence:

(i) The students would understand the importance of studying Calculus
(ii) They will gain a general idea of limits, continuity, derivatives and integration of a real Functions. Also, students will understand the basic notion of differential equations
(iii) Students analytical and reasoning skills will be improved, which ultimately enhance their thinking power.

Contents:

A brief history of the development of Calculus. Notion of variables and constants, idea of infinitesimals, Real Numbers and their properties, Intervals, Real Functions and their graphs. Monotone functions, even and odd functions, trigonometric functions. Limit of a Function and Limit Laws, Definition of a Limit, One-Sided Limits, Continuity, Continuity of important functions, Discontinuity. **[L-8H & T-4H]**

Differential Calculus: The Derivative as a Function, rules of differentiation, Differentiability, physical and geometrical significance of derivative, Derivatives of Trigonometric Functions, The Chain Rule.

Sign of the derivative and its significance. Local Maxima and minima of a real valued function.
[L-10H & T-5H]

Integral Calculus: Indefinite Integrals as anti-derivatives, standard results of Integration, The Definite Integral, properties of definite integrals, Integration as a limit of sum, Fundamental Theorem of Calculus, Area between curves. **[L-10H & T-5H]**

Ordinary Differential equation: Order & degree of a differential equation, Formation and solution of a differential equations, Differential equation of first order and first degree, simple applications.
[L-2H & T-1H]

Reading references:

Text Books:

1. Differential Calculus & Integral Calculus - R.K.Ghosh & K.C. Maity, (Books & Allied (P) Ltd).
2. An Introduction to Differential Equations - R.K.Ghosh & K.C. Maity, (Books & Allied (P) Ltd) (9th Edition).
3. Mathematics for Class 11 & 12 - S.N. Dey, (Chaya Prakashani).

Reference Books:

1. The History of the Calculus and Its Conceptual Development - Carl B. Boyer, (Dover Books on Mathematics).
2. Thomas' Calculus George B. Thomas , Joel Hass, Christopher Heil , Maurice D. Weir, Pearson .

SKILL ENHANCEMENT COURSES

Course Code: MATH3051
Course Name: Mathematical Modelling (Credit: 3, Marks: 50)
Total Hours: Lecture -30, Tutorial – 15

Objectives:

i. To provide fundamental concept of mathematical modelling,
ii. To discuss different types of models with the inclusion of linear, exponential, logistic, optimization, time series, simulation
iii. To discuss applicability of these models

Learning outcomes:

On successful completion of the course, the student will be well-conversed with the following outcomes

i. To gain knowledge about modelling
ii. To develop skill of model formation
iii. To update general competence

Knowledge:

i. Students to acquire basic knowledge concerning formation of various models
ii. Linear models help students to identify and estimate the relationship between variables, to analyze trends, to predict and make decisions from outcomes
iii. Exponential models help students to comprehend the rapid and often accelerating changes that occur in diverse natural and social systems
iv. Logistic model concerning real-world problems promote students to understand the limitations and saturation points of various processes
v. Optimization models empower students to take optimal decision and maximize the desired outcomes while considering real-world limitations and constraints
vi. Probabilistic/Stochastic models help students to handle uncertainty and make reasonable decisions by quantifying the likelihood of different outcomes
vii. Time series models facilitate students to analyze data, identify patterns, and make accurate predictions crucial for forecasting and understanding trends
viii. Simulation models provide powerful approach to study those systems in the event of non-availability of analytical solutions, support performance evaluation, risk analysis and decision support

Skill: Students to be

i. exposed to various mathematical models and their real-life applications
ii. benefited in simulations, understanding and predicting complex systems.

General competence:

i. To empower students the to understand the construction/framing mathematical models
ii. To analyze and solve the real-world problems mathematically
iii. To employ the usage of mathematical tools and techniques for the outcomes of those problems

Contents:

Overview of mathematical modelling and its applications in understanding real-world phenomena. Introduction to model classifications (Deterministic, Stochastic, Continuous, Discrete); Linear models and their applications; Usage of linear regression for modelling relationships between variables; Fitting linear models to data in analyzing trends and making predictions; Exponential model and its applications; Usage of exponential growth and decay models in population studies, finance, compound interest, half-life, and other relevant fields. **[L-8H & T-4H]**

Logistic models and their applications; Usage of logistic growth models in population studies, ecology, and epidemiology; Significance of logistic models in situations where growth is initially rapid but levels off over time. Optimization models and their applications; Use of linear programming and optimization techniques to maximize or minimize objectives; Importance of optimization models in resource allocation, production planning, and decision-making. Probabilistic/Stochastic models and their applications. **[L-12H & T-6H]**

Time series models and their applications; Importance of time series models in analyzing trends, seasonality, and forecasting future outcomes with applications. Introduction to simulation models and their applications; Monte Carlo simulation model, simulating deterministic features (area under a curve, volume under a surface) and other techniques for modelling uncertainty; Significance of simulation models in evaluating performance, risk analysis, decision support, random number generation. **[L-10H & T-5H]**

Suggested Books:

Text Books:

1. Mathematical Modeling: Models, Analysis, and Applications, Sandip Banerjee, Chapman and Hall/CRC.

2. A First Course in Mathematical Modeling, Frank R. Giordano, William P. Fox, and Steven B. Horton, Brooks/Cole.
3. Mathematical Models in Biology: An Introduction, Elizabeth S. Allman and John A. Rhodes, Cambridge University Press.
4. Practical Applied Mathematics: Modelling, Analysis, Approximation, Sam Howison, Cambridge University Press.

Reference Books:

1. Modelling with Mathematics: Authentic Problem Solving in Middle School, Nancy Butler Wolf, Heinemann.
2. Mathematical modelling, J. N. Kapur, New Age International Private Limited
3. Mathematical Modeling: Applications with GeoGebra, Jonas Hall and Thomas Lingefjärd, Wiley.
4. Mathematical Modeling, Mark. M. Meerschaert, Academic Press Inc.
5. Differential Equations and their applications, Zafar Ahsan, Prentice Hall India Learning Private Limited.

SEMESTER – IV

MAJOR COURSES

Course Code: MATH4011
Course Name: Metric Spaces (Credit: 5, Marks: 75)
Total Hours: Lecture -60, Tutorial – 15

Objectives:

Metric space is an indispensable intermediate in course of evolution of the general topological spaces. It generalizes the idea of distance between two points on the real line. In mathematics, a metric space is a set together with a distance. The distance is measured by a function called a metric or distance function. Metric spaces are the most general setting for studying many of the concepts of mathematical analysis and geometry.

Learning outcomes:

Students will be able to understand and appreciate the concept of a metric space by recognizing suitable examples. Students will be familiar with the fundamental notions of continuity, convergence and properties of completeness and compactness in a metric space.

Knowledge: Students will

i. be able to understand the distance function over the Euclidean spaces, space of all real valued continuous functions, sequence spaces etc.
ii. be able to learn the geometrical meaning of each of the metric properties.
iii. be able to classify the notion of open and closed balls for a given metric space.
iv. get exposure to the concept of continuity of functions.
v. learn the convergence of a sequence, the Cauchyness of a sequence in a given metric space.
vi. get exposure to the general notion of compactness property on a metric space and its analogue results in classical real and complex analysis.

Skills: Students would be

i. able to study the metric properties on a given metric space.
ii. able to study the topological properties of a metric space.
iii. motivated to work out various problems independently on the allied topics.
iv. influenced to study the analogue properties of a metric space in the space of real and complex numbers.

General Competence:

4. It helps the students to read and to learn further topics in analysis.
5. It motivates the students to make easier at understanding the use of functional analysis in applied problems.

Contents:

Metric spaces: Definition and examples. Open and closed balls, neighbourhood, interior points, open sets, interior of a set. limit points, closed sets, closure of a set, diameter of a set, boundedness of a set, exterior points, frontier points, boundary points, metric subspaces, equivalent metrics. **[L-8H & T-2H]**

Convergence of a sequence , Cauchy sequences, bounded sequences, complete metric spaces, dense sets, nowhere dense sets , sets of first and second category, Baire's category theorem, Cantor's intersection theorem, completion of a metric space, completeness property of $\mathbb{R}^n$, $C[a,b]$ with sup metric, l_p $(1 \leq p < \infty)$, incompleteness property of l_∞ and $C[a,b]$ with integral metric. **[L-12H & T-3H]**

Limit and continuity of mappings defined on metric spaces, sequential criterion of continuity, uniform continuity, homeomorphism, contraction mapping, Banach's Contraction Principle and its applications, viz. existence theorem on ODE (Picard's theorem), implicit function theorem, Fredholm integral equation, solution of a system of linear equations. **[L-8H & T-2H]**

Separated sets, connected sets, connectedness of a metric space and its properties, connectedness property under continuity, connected subsets of $\mathbb{R}$, components and relevant theorems. **[L-8H & T-2H]**

Open cover, compactness, countable compactness, sequential compactness, B-W compactness property, ϵ-net, totally bounded sets, coherence between compactness, completeness and totally boundedness property, Lebesgue number, Lebesgue covering lemma, equivalence of compactness, countable compactness, sequential compactness and B-W compactness property. Finite intersection property, compactness property using finite intersection property, compactness property under continuity and uniform continuity.

[L-16H & T-4H]

First and second countability of a metric space, separability and Lindelöf properties of a metric space.

[L-8H & T-2H]

Suggested Books:

Text books:

1. B. K. Lahiri, Elements of Functional Analysis, World Press, 1992.
2. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
3. S. Kumaresan, Topology of Metric Spaces, Narosa Publishing House, 2006.

Reference Books:

1. E. T. Copson, Metric spaces. Cambridge University Press,1968.
2. D. Gopal, A. Deshmukh, S. Ranadive, and S. Yadav, An introduction to Metric Spaces, Chapman & Hall, 2022.
3. A. K. Banerjee and A. Dey, Metric Spaces and Complex Analysis, New Age International Publishers, 2008.
4. B. Garai, An Introduction to Metric Spaces and Functional Analysis, Books and Allied, 2020.

Course Code: MATH4012

Course Name: Group Theory & Ring Theory (Credit: 5, Marks: 75)

Total Hours: Lecture -60, Tutorial – 15

Objectives:

Group and Ring are pivotal and initial steps to the learning modern algebra. Therefore, in this semester full stress on group and ring theory are implemented.

Learning outcomes:

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. group theory which is enough for a student to appear at different competitive examination within India and abroad.
ii. ring theory which almost covers its basic areas that helps students to grasp advanced areas related to this subject by themselves.
iii. its wide applicability in different branch of sciences.

Skills: The students would be able to

i. understand the beauty of structures and structure preserving maps.
ii. simplify a mathematical problem in different field of science using group and ring theory.
iii. initiate tricks of action of groups on a set or set with one or more structures to crack intricated problems.
iv. identify nature of a groups, specifically finite or finitely generated abelian groups.

General competence: The students would gain

i. descriptive idea of group and ring theory.
ii. to properly analyze algebraic properties of ring of integers.
iii. knowledge of loss and gain in generalizing the algebraic concept of integers.
iv. of understanding categorical similarities of structures and their commonness in properties.
v. expertized in solving many tricky problems in group and ring theory.

Contents:

Group: Homomorphism, isomorphism, endomorphism, automorphism, inner automorphism, Quotient group, Isomorphism theorems (1st, 2nd and 3rd), Correspondence theorem, Normalizer of a set, Commutator subgroup, abelianization of a group-universal property and uniqueness, maximal normal subgroup, simple group, Dihedral group of order n and Quaternion group – their properties, Classification of all groups upto order 8, action of a group on a set - examples, representation of a group action in terms homomorphism, Cayley's theorem, stabilizer of a point and orbit of a point – their relation, free, faithful and transitive action, class equation, conjugacy class of an element, Burnside theorem, p-group and its properties (p prime), Cauchy's theorem on finite group, Sylow theorems (1st, 2nd, 3rd) – its application, Direct product, Direct sum – their differences and properties, semi-direct product of two groups, Representation of finite abelian group. **[L-40H & T-10H]**

Ring: Ring homomorphism, quotient ring, isomorphism theorems (1st, 2nd and 3rd), correspondence theorem, maximal ideal, prime ideal and primary ideal - their existence, relations, properties. Radical of an ideal, Jacobson radical of a ring, nil ideal, nilpotent ideal, irreducible and prime elements, Euclidean domain, Principal ideal domain, unique factorization domain – their properties, polynomial rings of one indeterminate over a field F and integral domain, $F[x]$, irreducible criteria of polynomials. **[L-20H & T-5H]**

Suggested Books:

Text books:

1. Bhattacharyya P. B., Jain S.K, Nagpaul S.R, Basic Abstract Algebra, Cambridge University Press, 2nd Edition.
2. Dummit S. David, Foote M. Richard, Abstract Algebra, Willey Student Edition, Second Edition.
3. Malik D.S., Mordeson N. John, Sen M.K, Fundamentals of Abstract Algebra, The McGrow-Hill Companies.

Reference Books:

1. Serre J.P, Finite Groups: An Introduction, International Press of Boston Inc.
2. Maclane Saunders, Birkhoff Garrett, Algebra: Third Edition, AMS Chelsea Publishing.
3. Artin, M, Algebra, 2nd Edition, Pearson Education, India.
4. Fraleigh, J.B, A first course in Abstract Algebra, 7th Edition, Pearson Education, India.
5. Cohn P.M, Basic Algebra: Groups, Rings, and Fields, Springer; First Edition.
6. Gallian, Joseph, A., Contemporary Abstract Algebra, Cengage India Private Limited, Standard Edition.

Course Code: MATH4013

Course Name: Multivariate Calculus & Tensor Calculus (Credit: 5, Marks: 75)

Total Hours: Lecture -60, Tutorial – 15

Objectives:

To present the concepts of function of several variables, their calculus and related various properties and applications. Also, to present the concept of tensor algebra, tensor calculus and their properties

Learning outcomes:

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. functions of several variables, their calculus

ii. extrema of functions of n variables

iii. multiple integrals and their properties

iv. tensor calculus

Skills: The students would be able to

i. evaluate double limit, repeated limit etc. of functions of several variables,
ii. examine continuity of functions of several variables
iii. find partial and total derivatives of multivariate functions
iv. find extreme values of such functions, if they exist
v. calculate multiple integral of multivariate functions over certain domains, and so to find surface area and volumes of various shapes and bodies
vi. calculate various problems on tensor algebra and tensor calculus

General competence: The students would gain

i. general idea on limit, continuity, derivatives, integration of multivariate functions and general idea of tensors
ii. analytical and computing skills, which improve their visual and calculating powers.

Contents:

Multivariate Calculus (L-40H & T-10H)

Functions of several variables, repeated and double limits and continuity of functions of n variables. Partial derivatives, Euler's theorem, total derivative and differentiability, sufficient condition for differentiability. Chain rules, directional derivatives, Jacobian, the gradient, maximal and normal property of the gradient, tangent planes. Extrema of functions of n variables, method of Lagrange's undetermined multipliers, constrained optimization problems. **[L-20H & T-5H]**

Multiple integrals: Concept of double integral. Statement of existence theorem for continuous functions. Iterated or repeated integral, change of order of integration. Triple integral. Cylindrical and spherical coordinates. Change of variables in double integrals and triple integrals. Transformation of double and triple integrals. Determination of volume and surface area by multiple integrals. Differentiation under the integral sign, Leibniz's rule.

[L-20H & T-5H]

Tensor Calculus (L-20H & T-5H)

Historical study of tensor. Concept of E^n. Tensor as a generalization of vector in E^2, E^3 and E^n. Einstein's Summation convention. Kronecker delta. Algebra of tensor: Invariant, Contravariant and Covariant vectors. Contravariant, Covariant and mixed tensors. Symmetric and skew-symmetric tensors. Addition, subtraction and scalar multiplication of tensors. Outer product, inner product and contraction. Quotient law. **[L-8H & T-2H]**

Calculus of tensor: Riemannian space. Line element. Metric tensor. Reciprocal metric tensor. Raising and lowering of indices. Associated tensor. Magnitude of vector. Angle between two vectors. Christoffel symbols of different kinds and laws of transformations. Covariant differentiation. Gradient, divergence, curl and Laplacian. Ricci's theorem. Riemann-Christoffel curvature tensor. Ricci tensor. Scalar curvature. Einstein's space (Definition only). **[L-12H & T-3H]**

Suggested Books:

Text Books:

1. James Stewart, Multivariable Calculus, Concepts and Contexts, 2nd Ed., Brooks /Cole, Thomson Learning, USA, 2001 18
2. B. Spain, Tensor Calculus: A Concise Course, Dover Publications, 2003

Reference Books:

1. Horst R. Beyer, Calculus and Analysis, Wiley, 2010.
2. G.B. Thomas and R.L. Finney, Calculus, 9th Ed., Pearson Education, Delhi, 2005.
3. M.J. Strauss, G.L. Bradley and K. J. Smith, Calculus, 3rd Ed., Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
4. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer (SIE), 2005.
5. S. N. Mukhopadhyay and S. Mitra, Mathematical Analysis, Volume – II, U. N. Dhur & Sons Pvt. Ltd., 2014
6. T. Apostol, Mathematical Analysis, Narosa Publishing House.
7. Courant and John, Introduction to Calculus and Analysis, Vol II, Springer
8. W. Rudin, Principles of Mathematical Analysis, Tata McGraw-Hill.
9. I. S. Sokolnikoff, Tensor Analysis: Theory and Applications, John Wiley and Sons, Inc., New York, 1951.
10. M. C. Chaki, A Text Book of Tensor Calculus, Calcutta Publishers, 2000.
11. U. C. De, A. A. Shaikh and J. Sengupta, Tensor Calculus, Alpha Science International Ltd; 2nd Revised Edition, 2007.

MINOR COURSES

Course Code: MATH4021
Course Name: Ordinary Differential Equations (Credit: 4, Marks: 75)
Total Hours: Lecture -45, Tutorial – 15

Objectives:

To study ordinary differential equations through analytic and qualitative approaches.

Learning outcomes:

On completion of the course, the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Knowledge: The students would gain knowledge about

i. qualitative analysis of the ordinary differential equations.
ii. use of ordinary differential equations in different areas of mathematics.

Skills: The students would be able to

i. apply the solution techniques of the ordinary differential equations in different physical problems.
ii. solve the ordinary differential equations in different methods.
iii. apply the ordinary differential equations in different areas.

General competence: The students would gain

i. general idea about the solution techniques of ordinary differential equations.
ii. the distinct features of various types of ordinary differential equations.
iii. experience to solve differential equations using analytical approach.

Contents:

Picard's existence theorem (statement only) for $\frac{dy}{dx} = f(x, y)$ with $y = y_0, x = x_0$. Exact differential equations, condition of integrability. Equation of first order and first degree-exact equations and those reducible to exact form. Equations of first order higher degree-equations solvable for $p = \frac{dy}{dx}$, equations solvable for y, equations solvable for x, singular solutions, Clairaut's form. Singular solution as envelope to family of general solution to the equation.

[L-15H & T-5H]

Linear differential equations of second and higher order. Two linearly independent solutions of second order linear differential equation and Wronskian, general solution of second order linear differential equation, solution of linear differential equation of second order with constant coefficients. Particular integral (P.I.) for second order linear differential equation with constant coefficients for polynomial, sine, cosine, exponential function and for function as combination of them or involving them. Method of variation of parameters for P.I. of linear differential equation of second order. Homogeneous linear equation of n-th order with constant coefficients. Reduction of order of linear differential equation of second order when one solution is known. **[L-18H & T-6H]**

Simultaneous linear ordinary differential equation in two dependent variables. Solution of simultaneous equations of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Equation of the form (Paffian form) $Pdx + Qdy + Rdz = 0$. Necessary and sufficient condition for existence of integrals of the above. Qualitative studies of differential equations, Equilibrium points and their classifications, Phase plane analysis, Plotting of phase diagrams for some simple problems.

[L-12H & T-4H]

Suggested Books:

Text Books:

1. D. Murray, Introductory Course in Differential Equations, Longmans Green and Co, 1897.
2. E.A. Coddington, N. Levinson, Theory of Ordinary Differential Equations, McGraw Hill, New York, 1955.
3. S.L. Ross, Differential Equations, 3rd Ed., John Wiley and Sons, India, 2004.

Reference Books:

1. Boyce and Diprima, Elementary Differential Equations and Boundary Value Problems, Wiley, 2012.

2. G. F. Simmons, Differential Equations with Applications and Historical Notes, 2nd edition, McGraw Hill Education, 2017.

3. G. C. Layek, An Introduction to Dynamical Systems and Chaos, 2nd Edition, University Texts in the Mathematical Sciences, Springer, Singapore, 2024.